RESPONSE TO VIBRATIONAL DISTURBANCE OF THE MAGNET FOUNDATION

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Assumptions:

- Soil under the concrete slab of uniform density and elasticity (constant p and constant modulus of elasticity E).
- Constant frictional damping. 2.
- No coupling between vertical and horizontal motion. 3.

Vertical Motion

Hooke's law:
$$\frac{z-z_0}{z_0} = \frac{\text{Tension}}{E} = \frac{\text{Force}}{AE}$$

A = area of the slab

Force:
$$F = -Mg - M\frac{d^2z}{dt^2} - F_f + F_d$$

g = gravity acceleration F_f = frictional force $C \frac{dz}{dt}$

F_d = driving force (external and/or internal)

= f cos wt

 $= M_1 + A(L - z_0)\rho$

M₁ = mass of concrete slab + magnets

= distance of slab to bedrock (assumed constant)

ρ = density of soil

Substitution of the force F in Hooke's law gives

$$\frac{z - z_0}{z_0} = -\frac{Mg - M \frac{d^2z}{dt^2} - C \frac{dz}{dt} + f \cos wt}{AE}$$

Setting
$$z - z_0 + \frac{Mgz_0}{AE} = y$$
, $\frac{dz}{dt} = \frac{dy}{dt}$ and

$$\frac{d^2z}{dt^2} = \frac{d^2y}{dt^2}$$
 we obtain

for
$$z_0 = L (M = M_1)$$

$$\frac{d^2y}{dt^2} + \lambda \frac{dy}{dt} + \Omega^2 y = \varepsilon \cos \omega t.$$

where
$$\lambda = \frac{C}{M_1}$$
, $\Omega^2 = \frac{AE}{M_1L}$ and $\epsilon = \frac{f}{M_1}$.

The solution of this equation can be written in the form

$$y = \frac{\varepsilon}{\sqrt{(\Omega^2 - \omega^2)^2 + (\lambda \omega)^2}} \cos (\omega t - \phi), \tan \phi = \frac{\lambda \omega}{\Omega^2 - \omega^2}$$

We see that at resonance ($\omega = \Omega$), the amplitude is limited only by the frictional damping (λ). Knowing E, L, f and λ will give a rough estimate of the effects of vibrational disturbances.

In the foregoing treatment, the soil underneath the foundation is approximated by a pile. A more realistic result may be obtained by considering the soil as an "elastic" half-space. The weight of the concrete slab and the magnets will compress the soil and introduce vertical and horizontal strain. The static displacement is given by

$$\Delta z = \frac{M_1 g(1 - \mu)}{4G\sqrt{A/\pi}}$$

Here, μ is the Poisson's ratio $\frac{\text{horizontal strain}}{\text{vertical strain}}$ and $G = \frac{E}{2(1+\mu)}$ in the shear modulus.

Comparing Δz with the corresponding displacement for the pile approximation $\frac{M_1\,g\ell}{AE}$ we obtain the effective spring constant $\frac{4G}{1-\mu}\,\sqrt{\frac{A}{\pi}}$ and the resonant frequency $\Omega^2=\,\frac{4G}{M_1\,(1-\mu)}\,\sqrt{\frac{A}{\pi}}$.

To estimate the damping, we assume that the damping constant C is proportional to the area A, the density ρ and the wave velocity $\sqrt{\frac{\bar{G}}{\rho}}$. To make the ratio:

spring constant independent of the Poisson's ratio, we set

$$C = \frac{A\sqrt{G\rho}}{1-\mu} \text{ or } \lambda = \frac{A\sqrt{G\rho}}{M_1(1-\mu)} \ .$$

Assuming: $\rho = 2 \times 10^3 \text{ kg/m}^3$

$$G = 400 \text{ kg/cm}^2 \approx 4 \times 10^7 \text{ N/m}^2$$

$$A = 5 m^2$$

$$M_1 = 1.2 \times 10^4 \text{ kg}$$

$$\mu = 0.3$$

we obtain $\Omega = 155 \text{ sec}^{-1} \text{ (F } \approx 25 \text{ Hz)}$

$$\lambda = 168 \text{ sec}^{-1}$$

Substitutions of Ω and λ in $A_y = \frac{\varepsilon}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + \left(\lambda \omega\right)^2}}$ we find at resonance

$$A_y = 4 \mu m \text{ for } \epsilon = 0.01 \text{ g} \approx 0.1 \text{ m/sec}^2$$
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